$\Rightarrow f'(x) = 1 + f^2(x)$

$\forall x \in [0,1]$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

$$= \frac{1}{1 + f^2(f^{-1}(x))}$$

$$= \frac{1}{1+u^2}$$

donc

$\forall u \in [0,1]$, $\boxed{(f^{-1})'(x) = \frac{1}{1+u^2}}$

c Calculer $\int_0^1 \frac{dx}{1+x^2}$

$$\int_0^1 \frac{du}{1+u^2} = \int_0^1 (f^{-1})'(x)$$

$$= [f^{-1}(x)]_0^1$$

$$= f^{-1}(1) - f^{-1}(0) = \frac{\pi}{4} - 0 = \boxed{\frac{\pi}{4}}$$